CHAPTER 3

KINEMATICS IN TWO DIMENSIONS; VECTORS

OBJECTIVES

After studying the material of this chapter, the student should be able to:

1. Represent the magnitude and direction of a vector using a protractor and ruler.
2. Multiply or divide a vector quantity by a scalar quantity.
3. Use the methods of graphical analysis to determine the magnitude and direction of the vector resultant in problems involving vector addition or subtraction of two or more vector quantities. The graphical methods to be used are the parallelogram method and the tip to tail method.
4. Use the trigonometric component method to resolve a vector components in the x and y directions.
5. Use the trigonometric component method to determine the vector resultant in problems involving vector addition or subtraction of two or more vector quantities.
6. Use the kinematics equations of Chapter two along with the vector component method of Chapter three to solve problems involving two-dimensional motion of projectiles.

KEY TERMS AND PHRASES

scale factor
vector algebra
resultant
tip to tail method
parallelogram method
trigonometric component method
horizontal component
vertical component
projectile motion
relative velocity

CONCEPTS AND EQUATIONS

ADDITION OF VECTORS—GRAPHICAL METHODS

If two vectors are in the same direction and along the same line, the RESULTANT VECTOR will be the arithmetic sum of the magnitudes and the direction will be in the direction of the original vectors. If the vectors are in opposite directions and along the same line, the resultant vector will have a magnitude equal to the difference in the magnitudes of the original vectors and the direction of the resultant will be in the direction of the vector that has the greater magnitude.

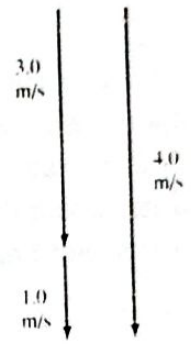
If the vectors are at some angle to each other than 0° or 180°, then special methods must be used to determine the magnitude and direction of the resultant. Two graphical methods used are the (1) TIP TO TAIL METHOD and (2) PARALLELOGRAM METHOD.

The graphical methods involve the use of a ruler and a protractor. An appropriate SCALE FACTOR must be used to represent the vectors. For example, a velocity vector that has a magnitude of 3.0 m/s and is directed due east can be represented by a line 3.0 cm long directed along the +x axis.

3.0 cm +x

TIP TO TAIL METHOD

The tip to tail method can be conveniently used for two or more vectors. The method consists in moving the vectors parallel to their original direction until the tail of one vector is at the tip of the next vector. Once all of the vectors are joined in this manner the resultant vector can be determined. The resultant is drawn from the tail of the first vector to the head of the last vector. For example, the resultant velocity (v_R) relative to the bank of a river for a person swimming at 3.0 m/s downstream in a river with a current of 1.0 m/s can be determined as shown in the figure. The magnitude of the resultant can be determined by measuring the length of the vector and multiplying by the scale factor. The scale used in the diagram is 1.0 cm = 1.0 m/s.



Using a ruler it can be determined that the resultant has a length of 4.0 cm. Therefore, the magnitude is (4.0 cm) x (1.0 m/s)/(1.0 cm) = 4.0 m/s. The direction of the resultant vector is downstream.

PARALLELOGRAM METHOD

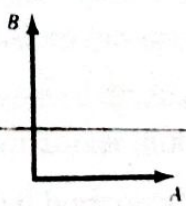
The PARALLELOGRAM METHOD is useful if two vectors are to be added. If more than two vectors are involved, the method becomes cumbersome and an alternate method should be used.

The two vectors are drawn to scale and joined at the tails. Dotted lines are then drawn from the tip of each vector parallel to the other vector. A protractor can be used to insure that the lines are drawn parallel. The finished diagram is a parallelo-gram. The resultant is along the diagonal of the parallelogram and extends from the point where the tails of the original vectors touch to the point where the dotted lines cross. As in the tip to tail method, the resultant is determined by measuring the length of the resultant and multiplying by the scale factor used to represent the vectors. The angle of the resultant above (or below) the x-axis is determined by using a protractor.

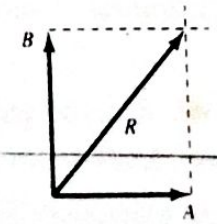
For example, determine the sum of the following vectors:

v_1 = 3.0 m/s due east, v_2 = 4.0 m/s due north.

step 1. Draw a diagram representing each vector with the tail of each joined at a point. Scale: let 1.0 cm = 1.0 m/s



step 2. Complete the parallelogram. Draw the resultant across the diagonal. Determine v_R.



Using a ruler, it can be determined that the resultant has a length of 5.0 cm. Therefore, the magnitude of the resultant is (5.0 cm) x (1.0 m/s)/(1.0 cm) = 5.0 m/s. Using a protractor, the angle θ can be determined to be 53° north of east. Therefore, v_R = 5.0 m/s ∠53° N of E.

SUBTRACTION OF VECTORS

The negative of a vector is a vector of the same magnitude but in the opposite direction. Thus, if vector v = 5 m/s due east, then -v = 5 m/s due west.

In order to subtract one vector from another, rewrite the problem so that the rules of vector addition can be applied. For example, A - B can be rewritten as A + (-B). Determine the magnitude and direction of -B and apply the rules of vector addition to solve for the resultant vector.

MULTIPLICATION OF A VECTOR BY A SCALER

The product of a vector times a scalar has the same direction as the vector and a magnitude equal to the product of the magnitude of the scalar times the magnitude of the vector. For example, if c is a scalar while V is a vector, then the product has a magnitude cV and the same direction as V. If c is a negative scalar, the magnitude of the resultant is still cV but the direction of the resultant is directly opposite that of V.

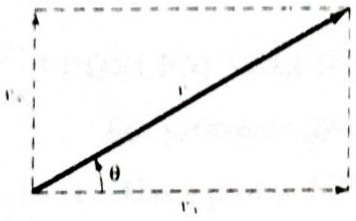
ANALYTIC METHOD FOR ADDING VECTORS

In the TRIGONOMETRIC COMPONENT METHOD, each of the original vectors is expressed as the vector sum of two other vectors. The two vectors are chosen to be in directions that are perpendicular to another one. At first, this method may seem long and tedious. However, with some practice this method is by far the most useful for this course.

For problems involving the addition of two or more vectors lying in the x-y plane, the method reduces to the following steps:

step 1. Resolve each vector into x and y components.

If the angle is measured from the x-axis to the vector, then the x component is equal to the product of the magnitude of the vector and the cosine of the angle. The y component is equal to the product of the magnitude of the vector and the sine of the angle. For example, for v = 5.0 m/s ∠30° N of E



$v_x = v \cos \theta = (5.0 \text{ m/s})(\cos 30°)$

then $v_x = (5.0 \text{ m/s})(0.87) = 4.3 \text{ m/s}$

$v_y = v \sin \theta = (5.0 \text{ m/s})(\sin 30°)$

then $v_y = (5.0 \text{ m/s})(0.50) = 2.5 \text{ m/s}$

step 2. Sign Convention

Assign a positive value to the magnitude if the component is in the +x or +y direction and a negative value if the component is in the -x or -y direction. Thus, for the vector used as an example in step 1,

v_x = + 4.3 m/s and v_y = + 2.5 m/s

step 3. Reduce the problem to the sum of two vectors.

Determine the sum of the x components (ΣX) and the sum of the sum of the y components (ΣY), where Σ is the upper case Greek letter sigma which is designated to mean "the sum of."

Since the x components are along the same line, their magnitudes are added arithmetically. This is also true for the y components.

step 4. Determine the magnitude and direction of the resultant.

Since ΣX and ΣY are at right angles, the Pythagorean theorem can be used to determine the magnitude of the resultant. The definition of the tangent of an angle can be used to determine the direction of the resultant.

RELATIVE VELOCITY

RELATIVE VELOCITY refers to the velocity of an object with respect to a particular frame of reference. As in Chapter 2, the reference frame is usually specified by using Cartesian coordinates, i.e. x, y and z axes, relative to which the position and/ or motion of an object can be determined. As stated in the text, the velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the relative velocity of the two reference frames, are known.

PROJECTILE MOTION

PROJECTILE MOTION is the motion of an object fired at an angle θ with the horizontal. This motion can be discussed by analyzing the HORIZONTAL COMPONENT of the object's motion independently of the VERTICAL COMPONENT of motion. If air resistance is negligible, then the horizontal component of motion does not change; thus $a_x = 0$ and $v_x = v_{xo}$ = constant.

The vertical component of motion is affected by gravity and is described by the equations for an object in free fall as discussed in Chapter 2. The following equations are used to describe the motion of a projectile.

vertical component of motion	horizontal component of motion
$v_y = v_{yo} - gt$	$X = v_{xo}\, t$
$Y = v_{yo}\, t - \frac{1}{2} g t^2$	and $v_x = v_{xo}$
$v^2 = v_{yo}^2 - 2 g Y$	

v_y is the vertical component of velocity at time t

v_{yo} is the initial vertical component of velocity, $v_{yo} = v_o \sin \theta$

g is the acceleration due to gravity, $g = 9.8\ m/s^2$

t is the time interval of the motion, $t_o = 0$ s

Y is the vertical displacement from the starting point. The starting point a reference point and the vertical displacement at this point is arbitrarily taken to be zero. Since $Y_o = 0$, Y_o does not appear in the equations.

X is the horizontal displacement from the starting point. X_o does not appear in the equation for horizontal motion because $X_o = 0$ at the starting point.

v_{xo} is the initial horizontal component of velocity. Assuming air resistance to be negligible, the horizontal component of velocity does not change during the motion. Therefore, $v_x = v_{xo} = v_o \cos \theta$.

SELECTED QUESTIONS WITH ANSWERS

QUESTION 1. Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?

ANSWER: Distance traveled is measured on the odometer while speed is measured on the speedometer. Distance and speed are scalar quantities while displacement and velocity are vector quantities. The odometer will record the distance traveled but does not record the displacement from the starting point. For example, if a student drives one mile to a convenience store and then back to his other starting point, the odometer will record a trip of two miles. However, the displacement from his or her starting point will be zero. Also, the speedometer records the magnitude of the motion but not the direction.

QUESTION 6. Two vectors have length $V_1 = 3.5$ km and $V_2 = 4.0$ km. What are the maximum and minimum magnitudes of their vector sum?

ANSWER: The maximum magnitude occurs if the two vectors are in the same direction. The resultant velocity is the arithmetic sum of their magnitudes, i.e. $V_R = 3.5$ km + 4.0 km = 7.5 km.

The minimum magnitude occurs if the two vectors are in opposite directions. The resultant velocity is the difference of their magnitudes, i.e. $V_R = 4.0$ km - 3.5 km = 0.5 km in the direction of the vector that has the greater magnitude.

QUESTION 12. A projectile has the least speed at what point in its path?

ANSWER: The motion of a projectile at any point in its motion is the vector sum of the horizontal component and the vertical component of its velocity. In the absence of air resistance, the magnitude of horizontal component does not change $v_x = v_{ox}$, As a projectile travels upward the magnitude of the vertical component decreases reaching zero at maximum height and on the way down the vertical component of velocity again increases. Since the velocity is the vector sum of the components, the magnitude of the velocity (i.e. the speed) must have a minimum value at the point where the vertical component is zero. Therefore, the minimum value of the speed occurs at the point where the projectile reaches maximum height.

PROBLEM SOLVING SKILLS

For problems involving vector addition or subtraction:

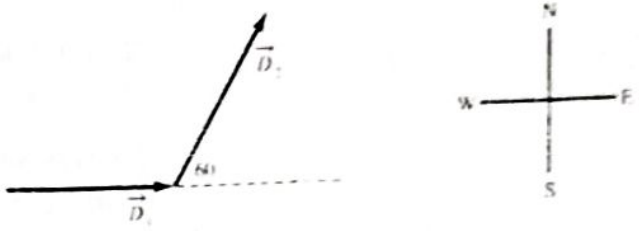
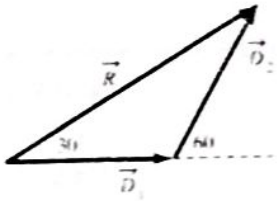
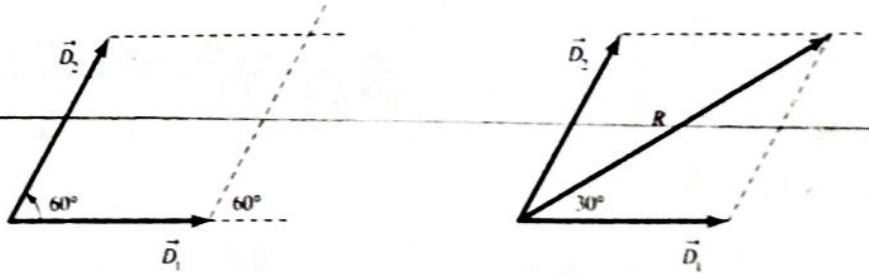
1. Use a protractor and ruler to accurately represent each vector involved in the problem. Make sure to use an appropriate scale factor in representing the vector.
2. Choose an appropriate method to solve the problem. If a graphical method is used be sure to measure both the magnitude and direction of the resultant.
3. If the trigonometric component method is used you must
   1) break each vector into x and y components.
   2) use the sign convention and assign a positive sign or a negative sign to the magnitude.
   3) determine the sum of the x components and repeat for the sum of the y components.
   4) use the Pythagorean theorem and simple trigonometry to solve for the magnitude and direction of the resultant.

For problems involving projectile motion:

1. Draw an accurate diagram showing the trajectory of the projectile.
2. Use the trigonometric component method to determine v_{ox} and v_{oy}.
3. Complete a data table using information both given and implied in the wording of the problem.
4. As in the free fall problems of Chapter two, use the appropriate sign convention depending on whether the object was initially moving upward or downward.
5. Memorize the formulas for projectile motion. It is also necessary to memorize the meaning of each symbol in each formula. Using the data from the completed data table, determine which formula or combination of formulas must be used to solve the problem.

PROGRAMMED PROBLEMS

PROBLEM 1. A man walks 50 meters due east, then 50 meters ∠60° north of east. Determine the magnitude and direction of the resultant displacement using the a) tip to tail method, b) parallelogram method and c) vector component method.

a. 1 Displacement is a vector quantity. Write each vector in vector notation.	Solution: (Sections 3-1 and 3-3) $\mathbf{D}_1$ = 50 m ∠due east $\mathbf{D}_2$ = 50 m ∠60° north of east
a. 2 Draw an accurate diagram locating each vector. Let the scale factor used in the drawing be 1.0 cm = 20 m. A protractor must be used to measure the angles.	
a. 3 Determine the magnitude and direction of the resultant vector using the tip to tail method.	This method consists in moving one of the vectors parallel to its original direction until its tail is at the tip of the other vector. The The resultant (R) is drawn from the unattached tail of the second vector to the unattached head of the first.  Based on careful measurement with a ruler, the magnitude of the resultant is 4.3 cm. Using the scale factor, the magnitude in meters can be determined. (4.3 cm)(20 m/1 cm) = 86 m The direction of the resultant is determined by use of a protractor and is 30° N of E, thus R = 86 m ∠30° N of E.
b. 1 Draw the vectors and complete the parallelogram.	The two vectors are drawn tail to tail. In order to complete the parallelogram, the dotted lines must be drawn parallel to the opposite sides. A protractor should be used to insure that this is done properly. 

b. 2 Use a protractor and a ruler to determine the magnitude and direction of the resultant. Note: remember to multiply by the scale factor.	The resultant is along the diagonal of the parallelogram. The direction is from the point where the tails are joined to the point where the dotted lines cross. As in the tip to tail method, the resultant is determined by measuring the length of **R** and multiplying by the scale factor. The direction is determined by using a protractor. The length of R is measured to be 4.3 cm; therefore, (4.3 cm)(20 m/1 cm) = 86 m. Using a protractor, the angle is measured to be 30°. Thus, **R** = 86 m ∠30° N of E

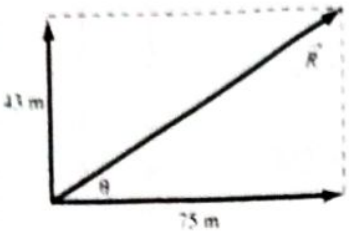
Vector Component Method: This method does not require an accurate diagram as part of the solution. However, a diagram reflecting the relative magnitudes and directions of the vectors is helpful.

c. 1 Make a drawing and resolve each vector into x and y components.	
c. 2 Use trigonometry to determine the magnitude of each component.	If the angle is measured from the east-west axis (x-axis) to the vector, then the x component is equal to the product of the magnitude of the vector and the cosine of the angle. The y-component is equal to the product of the magnitude of the vector and the sine of the angle. x-components $\mathbf{D}_{1x}$ = (50 m)(cos 0°) = + 50 m $\mathbf{D}_{2x}$ = (50 m)(cos 60°) = + 25 m y-components $\mathbf{D}_{1y}$ = (50 m)(sin 0°) = 0 $\mathbf{D}_{2y}$ = (50 m)(sin 60°) = +43 m
c. 3 Determine the sum of the x-components (Σ**X**) and the sum of the y-components (Σ**Y**).	The x components are in the +x direction; therefore, both are assigned positive values. The y-component of D_2 is in the +y direction; therefore, it has been assigned a positive value. $\Sigma\mathbf{X} = \mathbf{D}_{1x} + \mathbf{D}_{2x}$ = 50 m + 25 m = 75 m $\Sigma\mathbf{Y} = \mathbf{D}_{1y} + \mathbf{D}_{2y}$ = O m + 43 m = 43 m

c. 4

Complete the parallelogram using ΣX and ΣY as the vectors to be added. Solve for the magnitude and direction of the resultant vector.

Since ΣX and ΣY are at right angles, the Pythagorean theorem and trigonometry can be used to determine the magnitude and direction of the resultant.



$\mathbf{R} = [(\Sigma X)^2 + (\Sigma Y)^2]^{1/2}$

$= [(75\text{ m})^2 + (43\text{ m})^2]^{1/2}$

$\mathbf{R} = 86\text{ m}$

$\tan\theta = (43\text{ m})/(75\text{ m}) = 0.57, \theta = 30°$ N of E

thus, $\mathbf{R} = 86\text{ m} \angle 30°$ N of E

PROBLEM 2. Use the trigonometric component method to determine the sum of the following displacement vectors:

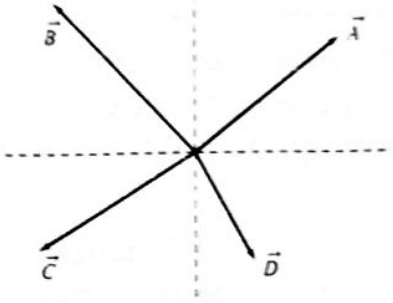
$\mathbf{A} = 5.0\text{ m} \angle 37°$ N of E, $\mathbf{B} = 6.0\text{ m} \angle 45°$ N of W,
$\mathbf{C} = 4.0\text{ m} \angle 30°$ S of W, $\mathbf{D} = 3.0\text{ m} \angle 60°$ S of E.

A. 1

Draw an accurate diagram with the tail of each vector located at the origin (x = 0, y = 0).

Solution: (Section 3-3)

Let 1.0 cm = 2.0 m



a. 2

Resolve each vector into x-components and y-components. Assign a positive sign to components directed toward the north and east and a negative sign to to components directed toward the south and west. Determine ΣX and ΣY.

$A_x = (5.0\text{ m})(\cos 37°) = +4.0\text{ m}$

$B_x = (6.0\text{ m})(\cos 45°) = -4.2\text{ m}$

$C_x = (4.0\text{ m})(\cos 30°) = -3.5\text{ m}$

$D_x = (3.0\text{ m})(\cos 60°) = +1.5\text{ m}$

$\Sigma X = -2.2\text{ m}$

$A_y = (5.0\text{ m})(\sin 37°) = +3.0\text{ m}$

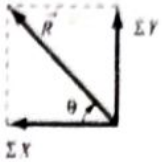
$B_y = (6.0\text{ m})(\sin 45°) = +4.2\text{ m}$

$C_y = (4.0\text{ m})(\sin 30°) = -2.0\text{ m}$

$D_y = (3.0\text{ m})(\sin 60°) = -2.6\text{ m}$

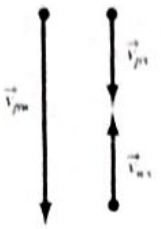
$\Sigma Y = +2.6\text{ m}$

a. 3 Use the Pythagorean theorem and trigonometry gonometry to determine the magnitude and direction of the resultant.	$R = [(\Sigma X)^2 + (\Sigma Y)^2]^{1/2}$ $= [(-2.2\ m)^2 + (2.6\ m)^2]^{1/2} = 3.4\ m$ $\tan \theta = (2.6\ m)/(-2.2\ m) = -1.2;\ \theta = 50°$ From trigonometry, the tangent of an angle located in the second or fourth quadrant has a negative value. It can be seen in the diagram that the vector lies in the second quadrant, i.e. north of west. Therefore R = 3.4 m ∠50° N of W

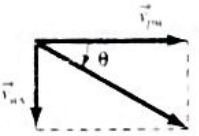


PROBLEM 3. The current in a river is 1.0 m/s. A woman swims 300 m downstream and then back to her starting point without stopping. If she can swim 2.0 m/s in still water, determine the time required for the round trip.

a. 1 Determine her velocity relative to the riverbank as she swims downstream.	Solution: (Sections 3-1 through 3-4) Let v_{pw} = velocity of the person relative to the river water v_{ws} = velocity of the water relative to the shore of the river, i.e. the river current v_{ps} = velocity of the person relative to the shore of the river
a. 1	The tip to tail method can be used to solve for v_{ps}. The vectors are along the same line; therefore, the resultant vector is the arithmetic sum of the magnitudes. 2.0 m/s 3.0 m/s 1.0 m/s $v_{ps} = v_{pw} + v_{ws}$ $= 2.0\ m/s + 1.0\ m/s$ $v_{ps} = 3.0\ m/s$ (downstream)
a. 2 Determine the time to swim downstream.	time = displacement/resultant velocity t = D/vps = (300 m)/(3.0 m/s) = 100 s

a. 3 Determine the time required to swim back to the starting point.	The tip to tail method can again be used to solve for vps. The woman is now swimming against the current. Thus  $v_{ps} = v_{pw} - v_{ws}$ $= v_{pw} + (-v_{ws})$ $= 2.0 \text{ m/s} + (-1/0 \text{ m/s})$ $v_{ps} = 1.0 \text{ m/s (upstream)}$ time = displacement/resultant velocity $t = D/v_{ps} = (300 \text{ m})/(1.0 \text{ m/s}) = 300 \text{ s}$
a. 4 Determine the total time for the round trip.	$t_{total} = t_{downstream} + t_{upstream}$ $t_{total} = 100 \text{ s} + 300 \text{ s} = 400 \text{ s}$

PROBLEM 4. The woman in the previous problem swims across the river to the opposite bank. If the river is 300 m wide, determine the time to swim across the river and back. She swims perpendicular to the river current and as a result she returns to a point downstream from her starting point.

a. 1 Determine the woman's velocity relative to the bank (vps).	Solution: (Sections 3-1 through 3-4)  The woman's velocity relative to the water (v_{pw}) is at right angles to the current (v_{ws}). The Pythagorean theorem can be used to solve for the woman's velocity relative to the river bank vps. $v_{ps} = [(v_{pw})^2 + (v_{ws})^2]^{1/2}$ $= [(2.0 \text{ m/s})^2 + (1.0 \text{ m/s})^2]^{1/2}$ $v_{ps} = 2.2 \text{ m/s}$ $\tan \theta = (1.0 \text{ m/s})/(2.0 \text{ m/s}) = 0.50, \theta = 27°$ $v_{ps} = 2.2 \text{ m/s} \angle 27°$

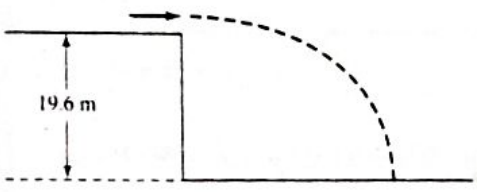
a. 2 Determine the time required to cross the river.	time = displacement/resultant velocity $t = D/v_{pw}$ but $\cos \theta = (300 \text{ m})/D$ and $D = (300 \text{ m})/(\cos 27°) = 340 \text{ m}$ $t = D/v_{pw} = (340 \text{ m})/(2.2 \text{ m/s}) = 150 \text{ s}$
a. 3 Determine the time required to swim back across the river and the total time for the round trip.	The return trip can be analyzed in like manner. The resultant velocity will be 2.2 m/s and the displacement will be 340 m. $t = D/v_{pw} = (340 \text{ m})/(2.2 \text{ m/s}) = 150 \text{ s}$ The total time for the round trip can now be determined. $t_{total} = t_{across} + t_{back}$ $t_{total} = 150 \text{ s} + 150 \text{ s} = 300 \text{ s}$

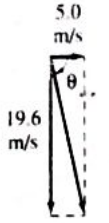
Alternate Method: Since the woman swims perpendicular to the current, the current does not affect the time for the round trip. An analogous situation is that of an airplane passenger walking from a window on the left side of the plane to a window on the right side of the plane. The time is the same if the plane is at rest or if it is moving at 450 miles per hour. Thus, time to cross river equals the distance across the river divided by the woman's velocity perpendicular to the river current.

$t = D/v_{pw} = (300 \text{ m})/(2.0 \text{ m/s}) = 150 \text{ s}$

$t_{total} = t_{across} + t_{back} = 150 \text{ s} + 150 \text{ s} = 300 \text{ s}$

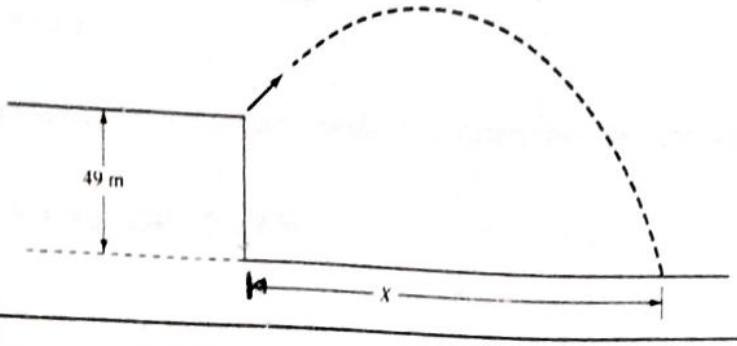
PROBLEM 5. A stone is thrown horizontally outward from the top of a bridge. The stone is released 19.6 meters above the street below. The initial velocity of the stone is 5.0 m/s. Determine the (a) total time that the stone is in the air an (b) magnitude and direction of the velocity of the projectile "just" before it strikes the street.

a. 1 Draw an accurate diagram showing the trajectory of the projectile.	Solution: (Section 3-6) 
a. 2 Determine the initial horizontal and vertical components of velocity.	Since the object is thrown horizontally, the initial vertical component (v_{yo}) is zero. The initial horizontal component (v_{xo}) is 5.0 m/s. Note: the vertical component of motion is independent of the horizontal motion.

a. 3 Complete a data table using information both given and implied in the problem.	Since the vertical component is downward, the downward direction will be designated as the positive direction in the same manner as the free fall problems discussed in Chapter 2. Therefore, the acceleration will be + g rather than - g. $v_{yo} = 0$ $\quad$ $t = ?$ $v_y = ?$ $\quad$ $X = ?$ $g = 9.8\ m/s^2$ $\quad$ $v_{xo} = 5.0\ m/s$ $Y = 19.6$
a. 4 Determine the total time that the stone is in the air.	$Y = \frac{1}{2} g t^2 + v_{yo} t$ $19.6\ m = \frac{1}{2} (9.8\ m/s^2) t^2 + (0\ m/s)\ t$ $t^2 = 2(19.6\ m)/(9.8\ m/s^2) = 4.0\ s^2$ either t = 2.0 s or - 2.0 s. Since the time of flight cannot be negative, the answer is t = 2.0 s.
b. 1 Determine the magnitude of the vertical component of the stone's velocity "just" before it hits the street.	$v_y = v_{yo} + g t$ $= 0\ m/s + (9.8\ m/s^2)(2.0\ s)$ $v_y = +\ 19.6\ m/s$
b. 2 Determine the magnitude and direction of the velocity of the projectile "just" before it strikes the street.	The horizontal velocity remains constant; therefore, both components of motion are now known. Use the Pythagorean theorem and trigonometry to determine the magnitude and direction of the resultant velocity (v_R).  $v_R = [(v_y) + (v_{xo})]^{1/2}$ $= [(19.6\ m/s)^2 + (5.0\ m/s)^2]^{1/2}$ $v_R = 20\ m/s$ $\tan \theta = v_y/v_{xo}$
	$\tan \theta = (19.6\ m/s)/(5.0\ m/s) = 3.9;\ \theta = 76°$ $v_R = 20\ m/s\ \angle 76°$ below the horizontal

PROBLEM 6. A projectile is fired with an initial speed of 113 m/s at an angle of 60° above the horizontal from the top of a cliff 49 m high. Determine the (a) time to reach maximum height, (b) maximum height above the base of the cliff reached by the projectile, (c) total time it is in the air, (d) horizontal range of the projectile.

Solution: (Section 3-6)

a. 1 Draw an accurate diagram showing the trajectory of the projectile.	
a. 2 Determine v_{oy} and v_{ox}.	$v_{yo} = v_o \sin 60° = (113 \text{ m/s})(0.87) = 98 \text{ m/s}$ $v_{xo} = v_o \cos 60° = (113 \text{ m/s})(0.50) = 57 \text{ m/s}$
a. 3 Complete a data table using information both given and implied.	$v_{yo} = 98 \text{ m/s}$ $\quad t = ?$ (to reach max height) $v_y = O$ (at maximum height) $a = -g = -9.8 \text{ m/s}^2$ (decelerates as it travels upward) $Y = ?$ (at max height) $v = 57 \text{ m/s}$ $\quad X = ?$
a. 4 Determine the time to reach maximum height.	$v_y = v_{yo} - gt$ $0 \text{ m/s} = 98 \text{ m/s} - (9.8 \text{ m/s}^2)\, t$ $t = 10 \text{ s}$
b. 1 Determine the maximum height above the cliff reached by the projectile. Now determine the maximum height above the ground reached by the projectile.	$Y = -\frac{1}{2} g t^2 + v_{yo} t$ $= -\frac{1}{2} (9.8 \text{ m/s}^2)(10 \text{ s})^2 + (98 \text{ m/s})(10 \text{ s})$ $= -490 \text{ m} + 980 \text{ m}$ $Y = 490 \text{ m}$ The total height above the base of the cliff can now be determined: $Y = 490 \text{ m} + 49 \text{ m} = 539 \text{ m}$

c. 1 Complete a data table and determine the time required for the projectile to fall from maximum height to the base of the cliff.	The downward direction is taken to be positive. Thus +g will be used instead of -g. $v_{yo} = 0$ (at maximum height) $t = ?$ $v_y = ?$ $Y = 539$ m $g = 9.8\ m/s^2$ $X = ?$ $v_{xo} = 57$ m/s $Y = \frac{1}{2} g t^2 + v_{yo} t$ $539\ m = 1/2\ (9.8\ m/s^2)\ t^2 + (0\ m/s)\ t$ $t^2 = 2(539\ m)/9.8\ m/s^2 = 110\ s^2$ $t = 11$ s
c. 2 Determine the total time the projectile is in the air.	total time = 10 s + 11 s = 21 s
d. 1 Use the equation for horizontal motion to determine the horizontal range.	$X = v_{xo} t = (57\ m/s)(21\ s)$ $X = 1200$ m

SUPPLEMENTARY PROBLEMS

PROBLEM 1. A man walks 3.0 km due east, then 1.0 km 45° north of east and finally 2.0 km 30°south of west. Use the tip to tail method, the parallelogram method, and the trigonometric component method to determine his final displacement from his starting point.

ANS. 2.0 km ∠8.4° south of east

PROBLEM 2. Use the trigonometric component method to determine the sum of the following displacement vectors:

A = 3.0 m ∠30° N of E, B = 4.0 m ∠45° N of W,

C = 5.0 m ∠53° S of W.

ANS. 3.0 m ∠63° N of W

PROBLEM 3. In a particular laboratory experiment, a spring gun placed on a table fires a steel ball horizontally outward. A student determines that the ball starts 1.0 m above the floor and travels 2.7 m horizontally before striking the floor. Determine the a) time that the ball is in the air and b) initial velocity of the ball.

ANS. a) 0.45 s b) 6.0 m/s

PROBLEM 4. The gun described in the previous problem was then arranged at a 45° angle above the horizontal and the student determines that the ball leaves the muzzle of the gun 1.1 m above the floor. Assuming that the initial velocity of the bullet remains the same, determine the a) total time that the ball is in the air and b) horizontal range of the projectile.

ANS. a) 1.1 s, b) 4.6 m